New Numerical Method for Radiation Heat Transfer in Nonhomogeneous Participating Media

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A new numerical method, called the YIX method, which solves the exact integral equations of distance-angular integration form for radiation transfer, is introduced in this paper. By constructing and prestoring the numerical integral formulae for the distance integral for appropriate kernel functions, this method eliminates the time consuming evaluations of the kernels of the space integrals in the formal computations. In addition, when the number of elements in the system is large, the resulting coefficient matrix is quite sparse. Thus, either considerable time or much storage can be saved. A weakness of the method is discussed, and some remedies are suggested. As illustrations, some one-dimensional and two-dimensional problems in both homogeneous and inhomogeneous emitting, absorbing, and linear anisotropic scattering media are studied. Some results are compared with available data.

Nomenclature

$D_3(t)$ = function defined by Eq. (4d)
e_g = radiative emissive power of medium
E_n = exponential integral of nth order
e_s = radiative emissive power of enclosure
H = characteristic length
k = extinction coefficient
$\boldsymbol{n}$ = inward unit normal vector at $\boldsymbol{r}$
P_i = constants defined by Eq. (7c)
Q_i = constants defined by Eq. (7b)
q_r = radiation flux vector
q_s = heat flux on boundary surface
$\boldsymbol{r}$ = position vector, (x,y)
$R(\omega)$ = distance from a point in medium to the nearest wall in direction ω
S_n = Bickley's function of nth order
t_i = constants [Eq. (4)]
u = optical coordinate defined by Eq. (12)
β = linear anisotropic scattering coefficient
ϵ = emissivity
θ = angular coordinate
λ = constant [Eq. (6)]
τ = optical depth
ω = albedo for scattering
$\boldsymbol{\omega}$ = direction vector

Introduction

RADIATION heat transfer in participating media is important in many applications such as the design of industrial furnaces, rocket combustion chambers, and novel high-temperature heat exchangers. A good numerical method that solves thermal radiation problems must be flexible enough to deal with complex geometries and real radiation properties, be able to deal with combined mode heat transfer problems, and be efficient so that not much computer time and storage is required. Unfortunately, almost no existing method satisfies all of those criteria. For example, the Monte Carlo method[1] is excellent in flexibility and requires little computer memory, but it can be very time consuming and inaccurate. The zonal method[2] and the finite element method[3,4] are both time and storage consuming, although they are very accurate and can be modified to deal with complex problems such as anisotropic scattering. The product integration method (PIM),[5] while faster than the zonal method and the finite element method, does not reduce the storage. The discrete ordinates method, developed to solve neutron transport problems,[6] was applied to radiation transfer.[7] It gives a sparse matrix so that the storage for large grid systems is less than for each of the above techniques except Monte Carlo, but it suffers from the ray effect[8] and requires large computer time to solve multidimensional combined mode heat transfer problems. The application of many methods using approximate formulations, e.g., the P-N method,[9] are restricted to certain circumstances because of their accuracy.

In this paper, a new numerical method, the YIX method, is presented to solve radiation problems in multidimensional emitting, absorbing, and anisotropic scattering media. The exact integral formulation for radiation transport[5] is used but in an alternative distance-angular integral form. Here, the method is described for a simple one-dimensional problem, extensions to a more complex case are considered, and example problems in two-dimensional participating media are solved to test the accuracy and efficiency of the method in one-dimension.

Description of the Method

In order to describe the main idea of the YIX method, a simple one-dimensional problem is chosen. Its extensions in two dimensions are given in the next section.

Consider in one-dimensional space a planar nonscattering and gray medium Ω bounded by two nonreflecting and black walls. It is assumed that no heat source or sink exists in the system and all radiation properties are constant.

Dividing Ω into N planar elements, the discretized equation of radiation transport using piecewise-constant interpolating

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functions in distance-angular integral formulation is[9]

$$2e_g(x_i) = \int_0^H kE_1(k|y - x_i|)e_g(y)\mathrm{d}y + e_{s1}E_2(kx_i) + e_{s2}E_2(k|H - x_i|), \qquad i = 1, \ldots, N \tag{1}$$

where e_g and e_s are the emissive powers in Ω and on the walls, respectively.

Both the zonal method and the product integration method calculate the elements of the coefficient matrices (the exchange areas in the zonal method) one by one and thus require many evaluations of the time-consuming kernel functions (the exponential integral for one-dimensional problems). However, the YIX method constructs integral formulas for the distance integral and stores the resulting abscissas to save much computing effort.

To do this, let us consider the following integral

$$I \equiv \int_0^L f(t)E_1(t)\mathrm{d}t \tag{2}$$

and subdivide it into

$$I = \int_{t_n}^L f(t)E_1(t)\mathrm{d}t + \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} f(t)E_1(t)\mathrm{d}t \tag{3}$$

where $0 = t_0 < t_1 < \ldots < t_n \leq L$, and t_i are constants to be decided.

Now, apply the following two-point approximation to the integral in each subregion.

$$I_i = \int_{t_{i-1}}^{t_i} f(t)E_1(t)\mathrm{d}t \approx a_i f(t_{i-1}) + b_i f(t) \tag{4a}$$

so that it is exact to the first order. This gives

$$a_i + b_i = \int_{t_{i-1}}^{t_i} E_1(t)\mathrm{d}t = E_2(t_{i-1}) - E_2(t_i)$$

and

$$a_i t_{i-1} + b_i t_i = \int_{t_{i-1}}^{t_i} tE_1(t)\mathrm{d}t = t_{i-1}E_2(t_{i-1}) - t_i E_2(t) + E_3(t_{i-1}) - E_3(t_i)$$

or, solving for a_i and b_i

$$a_i = E_2(t_{i-1}) - D_3(t_i) \tag{4b}$$

and

$$b_i = D_3(t_i) - E_2(t_i) \tag{4c}$$

where

$$D_3(t_i) \equiv \frac{E_3(t_{i-1}) - E_3(t_i)}{t_i - t_{i-1}} \tag{4d}$$

Substituting Eqs. (4) into Eq. (3) yields

$$I \approx [1 - D_3(t_1)]f(0) + \sum_{i=1}^{n-1}[D_3(t_i) - D_3(t_{i+1})]f(t_i) + [D_3(t_n) - D_3(L)]f(t_n) + [D_3(L) - E_2(L)]f(L) \tag{5}$$

Let

$$2[1 - D_3(t_1)] = [D_3(t_i) - D_3(t_{i+1})] \equiv \lambda = \text{const} \tag{6}$$

which allows the contribution of f to the integral to be the same at each integration point $t_i > 0$. This is desirable because it reduces the number of integrations as the distance increases (so the contribution of emissions at farther points is smaller).

When t_1 is known, λ and t_i ($i = 2, \ldots$) can be computed by solving Eq. (6) recursively until t_i is so large that $D_3(t_i) - D_3(\infty) = D_3(t_i) < \lambda$, i.e., t_{i+1} does not exist. Applying Eq. (6), Eq. (5) becomes

$$I = \lambda\left[\frac{1}{2}f(0) + \sum_{i=1}^{n-1} f(t_i)\right] + [D_3(t_n) - D_3(L)]f(t_n) + [D_3(L) - E_2(L)]f(L)$$

which greatly reduces the number of evaluations of the kernel functions. To completely eliminate the kernel function evaluations, the following approximation may be applied:

$$\int_{t_n}^L E_1(t)f(t)\mathrm{d}t \approx \frac{L - t_n}{t_{n+1} - t_n}\int_{t_n}^{t_{n+1}} E_1(t)f(t)\mathrm{d}t \approx \frac{L - t_n}{t_{n+1} - t_n}[E_2(t_n) - E_2(t_{n+1})]f(t_n), \qquad t_n \leq L < t_{n+1}$$

(In the case that t_{n+1} does not exist, the above integral is set to zero.) Note that this approximation is also to first-order and thus does not reduce the order of accuracy. The integral I now can be evaluated by

$$I = \lambda\left[\frac{1}{2}f(0) + \sum_{i=1}^{n-1} f(t_i)\right] + [P_{n+1} + LQ_{n+1}]f(t_n) \tag{7a}$$

where

$$Q_i \equiv \frac{E_2(t_{i-1}) - E_2(t_i)}{t_i - t_{i-1}} \tag{7b}$$

and

$$P_i \equiv D_3(t_{i-1}) - E_2(t_{i-1}) - t_{i-1}Q_i - \lambda \tag{7c}$$

When t_1 is known, all of the constants t_i, Q_i, and P_i can be computed and stored in advance. Therefore, in the computations of many integrals of the form of Eq. (2), the evaluation of the integral is replaced by a simple summation of f_i once the constants are calculated.

As an illustration of the distribution of t_i, Fig. 1 shows the distribution of t_i when $t_1 = 0.01$, which gives $n = 18$, $\lambda = 0.0553$, and $t_N = t_{18} = 2.805$, where t_N denotes the largest existing t_i.

Now the radiation transport Eq. (1) can be rewritten as

$$2e_g(x_j) = e_{s1}E_2(kx_j) + e_{s2}E_2(k|H - x_j|) + \int_0^{kx_j} E_1(t)e_g\left(x_j - \frac{t}{k}\right)\mathrm{d}t + \int_0^{k(H - x_j)} E_1(t)e_g\left(x_j + \frac{t}{k}\right)\mathrm{d}t \tag{8a}$$

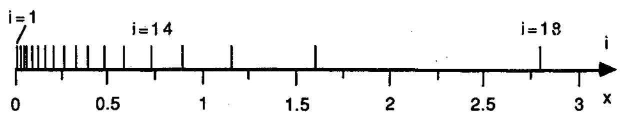

Fig. 1 Example of integration points for one-dimensional radiation, kernel = E_1.

Using Eq. (7a), the first integral is evaluated as

$$J \equiv \lambda\left[\frac{1}{2}e_g(x_j) + \sum_{i=1}^{n-1} e_g\left(x_j - \frac{t_i}{k}\right)\right] + [P_{n+1} + kx_jQ_{n+1}]e_g\left(x_j - \frac{t_n}{k}\right) \tag{8b}$$

In a computer program, one can compute the sum first by increasing i until $t_i > kx_j$; then the last term in Eq. (8b) is calculated. The second integral can be calculated in the same manner.

One difficulty in the use of the present technique is to decide in which element the integral point $x_j - t_i/k$ lies. If the space discretization is regular, the decision is easy; otherwise, especially for multidimensional problems, more work is needed. However, modification is possible to deal with irregular elements by using several regular subelements to approximate an irregular element.[10]

It should be pointed out that the discretized integral Eq. (1) can be solved in two ways. One method is by giving an initial guess of e_g; the right-hand side of Eq. (1) is computed by Eqs. (8a) and (8b) for every i to give a new e_g. This iterating procedure does not require any storage of coefficient matrices. It is, however, quite slow. Another way is to collect the coefficient matrices first, then solve the resulting algebraic equations either directly or iteratively. This method is much faster than the first at the expense of more storage. It should be noted, however, that the generation of the matrices is much faster than the product integration method and, unlike the zonal method or the product integration method, the matrix is quite sparse for problems with a large number of elements. This can be easily seen in Figs. 1 and 2 where many possible elements contain no integration point.

When $t_1 \to 0$, the resulting coefficient matrix (either in explicit form, as in the second method above, or in implicit form, as in the first method) should be identical to that given by the product integration method with exact integrations, i.e., in this limit, the results of the two methods should be identical.

Instead of a two-point integration scheme [Eq. (4a)] that is only linearly exact, higher-order formulas can be constructed. However, this seems unnecessary unless higher-order interpolating functions for e_g are used.

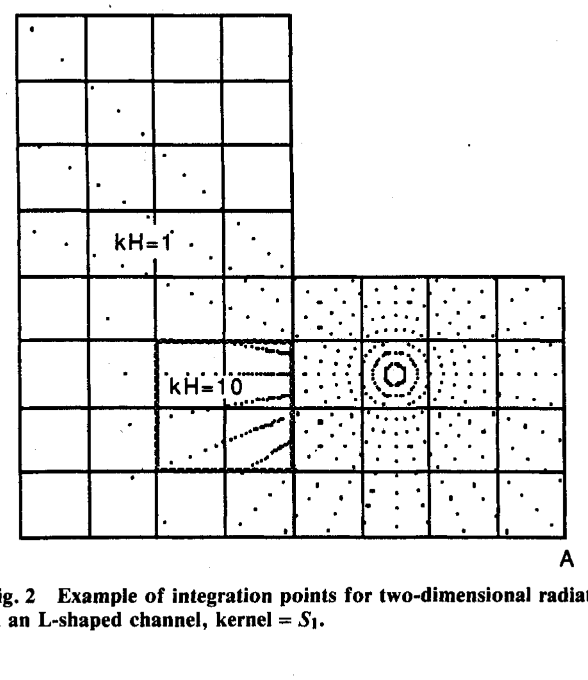

Fig. 2 Example of integration points for two-dimensional radiation in an L-shaped channel, kernel = S_1.

Extensions to Two-Dimensional Inhomogeneous Media

The method can be easily extended to more complex problems. As an illustration, let us consider in two-dimensional space an emitting, absorbing, and nonscattering medium, bounded by black walls. The walls may be obstructing, the radiation properties (here the absorption coefficient) of the medium may vary with position, and no heat source or sink will be considered. The radiation transport equation is[5,11]

$$4e_g(r) = \int_0^{2\pi} d\theta \int_0^{R(\theta)} \times S_1\left[\int_0^t k(r+\omega t')dt'\right]k(r+\omega t)e_g(r+\omega t)dt + \int_{\theta_0}^{\theta_0+\pi} S_2\left[\int_0^{R(\theta)} k(r+\omega t')dt'\right] \times e_s[r+\omega R(\theta)]\cos(-\omega, n')d\theta \tag{9}$$

where $\omega = (\cos\theta, \sin\theta)^t$, $R(\theta)$ is the length of the ray $r + \omega t$ from r to the intersecting point with the wall that first obstructs the ray, $n' = (-\sin\theta_0, \cos\theta_0)^t$ is the inward normal at $r' = r + \omega t'$, and S_i is Bickley's function defined by

$$S_n(x) = \frac{2}{\pi}\int_0^{\pi/2} e^{-x/\cos\theta} \cos^{n-1}\theta d\theta$$

Note that a similar distance-angular integration form was used in the finite element method[3] to subtract singularities.

In the same manner as in the one-dimensional case, we can construct integral formulas for the distance integral of the form

$$I = \int_0^L S_1(u)f(u)du = \lambda\left[\frac{1}{2}f(0) + \sum_{i=1}^{n-1} f(u_i)\right] + [P_{n+1} + LQ_{n+1}]f(u_n) \tag{10}$$

In fact, the procedure for u_i, Q_i, and P_i and the summation formula [cf. Eqs. (7a) and (8b)] in the two-dimensional case is exactly the same as in one dimension except that the function E_1 is replaced by S_1.

Approximating the angular integration in Eq. (9) numerically with uniformly distributed integration points, and utilizing the above formula, yields

$$4e_g(r) \approx \frac{1}{M}\sum_{k=1}^{M} \int_0^{R(\theta_k)} S_1\left[\int_0^t k(r+\omega_k t')dt'\right] \times k(r+\omega_k t)e_g(r+\omega_k t)dt + \frac{1}{M^*}\sum_{k=1}^{M^*} \cos(-\omega_k n') \times S_2\left[\int_0^{R(\theta)} k(r+\omega_k t')dt'\right] e_s\left[r+\omega_k R(\theta)\right] d\theta \tag{11}$$

where θ_k are the integral points uniformly distributed in $[0, 2\pi]$ or $[\theta_0, \theta_0 + \pi]$, and M and M^* are the numbers of angular integration points in the medium and on the wall, respectively.

Letting

$$u = \int_0^t k(r+\omega_k t')dt' \tag{12}$$

the first integral of Eq. (11) becomes

$$\int_{u=0}^{t=R(\theta_k)} S_1(u)e_g(r+\omega_k t)du = \lambda\left[\frac{1}{2}e_g(r) + \sum_{i=1}^{n-1} e_g(r+\omega_k t_i)\right] + [P_{n+1} + LQ_{n+1}]e_g(r+\omega_k t_n) \tag{13}$$

Table 1 Comparative fluxes for one-dimensional planar problem

$\omega\beta$	−0.7				0.0				0.7			
kH	PIM	YIX	D&T	E	PIM	YIX	D&T	E	PIM	YIX	D&T	E
0.1	0.901	0.901	0.901	0.0	0.916	0.916	0.916	0.0	0.931	0.931	0.931	0.0
0.5	0.664	0.664	0.663	0.1	0.705	0.705	0.704	0.0	0.751	0.751	0.750	0.1
1.0	0.506	0.506	0.505	0.3	0.555	0.554	0.553	0.3	0.614	0.614	0.611	0.4
3.0	0.271	0.271	0.260	4.2	0.311	0.311	0.301	3.2	0.366	0.366	0.358	2.1

PIM = product integration method.[5] D&T = Dayan and Tien (Ref. 12).
E = percentage error between YIX and D&T.

where n satisfies $t_n \le R(\theta_k) < t_{n+1}$. To find the relationship between t_i and u_i, we apply the first-order finite difference to Eq. (12) to get

$$t_{i+1} = t_i + \frac{u_{i+1} - u_i}{k(r + \omega_k t_i)}$$

with $t_0 = u_0 = 0$. Since the values of u_i are known, t_i can be found recursively.

The distribution of the integration points in physical space varies according to the absorption coefficient distribution. In more highly absorbing regions, the integration points are more concentrated than in lower absorbing regions. Figure 2 shows a set of integration points in an L-shaped inhomogeneous medium. (The name of the method, YIX, was given simply because the three characters, I, Y, and X, look like the integration point distributions with 2, 3, and 4 angular points in the two-dimensional case.)

As for the one-dimensional case, both of the two ways mentioned in the last section can be used to solve Eq. (9).

Using corresponding formulations,[5] the YIX method can be directly extended to problems with reflections from walls and anisotropic scattering in the medium. Since more kernel functions appear for one-dimensional and two-dimensional problems, more sets of integration points t_i must be generated with respect to different t_1. Extension to a nongray medium should also be easy using a proper iteration strategy. By combining with a heat conduction or fluid flow code, the method can be used to solve combined radiation and conduction/convection problems.

Example Problems

The YIX method has been applied to several one-dimensional and two-dimensional problems. Here are some examples to show the efficiency and accuracy of the technique.

One-Dimensional Problem Between Infinite Parallel Plates: Homogeneous Medium

The same one-dimensional problem in planar emitting, absorbing, and linearly anisotropic scattering medium studied by Dayan and Tien[12] and recalculated with the product integration method by Tan[5] is solved by the present technique using eight space elements. The t_1 for the three kernels E_1, E_2, and E_3 are 0.001, 0.004, and 0.005, respectively. The heat fluxes, compared with the results of Tan[5] and of Dayan and Tien,[12] are given in Table 1.

It is found that the fluxes given by the YIX method and the product integration method are almost identical. This is not surprising because both methods use the same formulation and the same interpolations. The slight difference of radiation flux at $kH = 1$ and $\omega\beta = 0$ (where H is the thickness of the medium, ω the scattering albedo, and β the linear anisotropic scattering coefficient) is caused by the different numerical integration schemes of the YIX and the PIM. The results of the YIX method, like that of PIM, are excellent in comparison with Dayan and Tien's[12] exact solutions.

One-Dimensional Problem Between Infinite Parallel Plates: Inhomogeneous Medium

The geometry and boundary conditions are the same as above. Assume $H = 1$ and $\omega = 0$. The extinction coefficient k

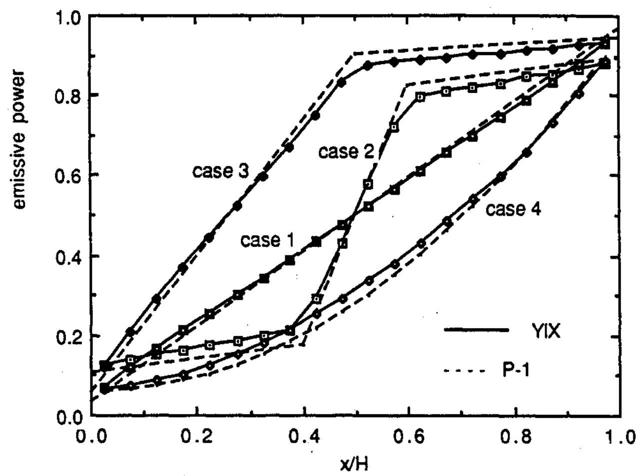

Fig. 3 Comparative dimensionless emissive power for one-dimensional radiation in planar media. Influence of extinction coefficient distribution.

is different in various regions. Four cases are considered:

1) $kH = 20$ in $0 < x/H < 1$.
2) $kH = 20$ in $0 < x/H < 0.5$, and $kH = 1$ in $0.5 < x/H < 1$.
3) $kH = 1$ in $0 < x/H < 0.4$, $kH = 20$ in $0.4 < x/H < 0.6$, and $kH = 1$ in $0.6 < x/H < 1$.
4) $kH = 20\, x/H$ in $0 < x/H < 1$.

In all cases, 20 elements are used. The same t_1 as above are chosen.

The results are shown in Fig. 3. Solutions using the P-1 approximation are also shown in the figure. Agreement of the results by the two methods is very good. Probably the small discrepancy is mainly due to the inaccuracy of the P-1 technique.

It was found (but not shown here), however, that the surface flux on a wall close against a highly absorbing region is poorly predicted, even though the emissive power distribution in the medium is accurately calculated. Note that this kind of error is also present in applying other methods based on the integral formulation (e.g., the zonal method and the product integration method; cf. Table 1 and the table in Ref. 5) and is caused by the assumption of constant emissive power in the element near the wall. When the optical thickness in this element is large, the contribution of this element to the flux on the neighboring surface is dominant; therefore, the constant emissive-power assumption would cause a large error if the integral formulation is used. In this case, a simple differential approximation, which is accurate in the optically thick region (e.g., the diffusion approximation), can be used to get an accurate value for surface flux. Another remedy for the problem would be applying higher-order interpolating functions to replace the piecewise constants that are used throughout this study.

This example shows that, as opposed to most methods based on the integral formulation, the YIX method can deal with inhomogeneous media as easily as a homogeneous one. This enables us, after straightforward modifications, to use the band model to take real-gas properties into consideration.

Depending on which of the two solution methods mentioned before is used, two corresponding iterating methods for real-gas problems are possible. In the first method, the optical properties (absorption coefficient and/or scattering coefficient) are renewed in each iteration as soon as a new emissive power value is obtained. In the second method, the optical property distribution is first guessed, based on the initial emissive power (or, equivalently, temperature) distribution, to generate the coefficient matrices; the emissive power distribution is then solved to give a new optical property distribution. Iteration continues until a convergence criterion is satisfied.

In the real-gas case, the efficiency of the first solution method will be unaffected compared with the constant optical property case; while the efficiency of the second method will be reduced by the factor of the number of iterations. It is difficult to tell, however, which one is superior to the other.

Two-Dimensional Problem in a Square Enclosure: Homogeneous Medium

The problem of two-dimensional radiation heat transfer in a two-dimensional absorbing, emitting, and linearly scattering square medium studied by Tan,[5] who used the PIM, is calculated by the YIX method for comparison. Black walls and no heat source or sink in the medium are assumed. The geometry and boundary conditions are shown in Fig. 4. In this example, 8×8 grid points in the medium are used. Other grids (e.g., 5×5 and 20×20) were also tried, but no significant difference was found.

Figure 5 depicts the comparative hot (lower) wall heat fluxes influenced by the anisotropic scattering parameter $\omega\beta$. As expected, excellent agreement is found between the YIX method and the PIM.

Crosbie and Schrenker[13] treated the pure-scattering case in a two-dimensional geometry. They used a singularity subtraction technique as was done here, but used Gaussian quadrature for evaluation of the integrals in the source function equation rather than the YIX treatment presented in this work. The agreement of the YIX results with the calculations of Crosbie and Schrenker[13] in the nonscattering case is also good (Fig. 5). It should be noted, however, that the data in the figure are not symmetric with respect to the vertical centerline ($x/H = 0.5$), despite having symmetric nodes and integral points. This asymmetry is caused by the YIX method itself and the cause is somewhat similar to the ray effect of the discrete ordinates method.[8] To understand this phenomenon, consider point A in Fig. 2. When the boundary conditions about A change smoothly, then the integral point A well represents its neighborhood and causes no computational problem. However, if the boundary condition about this point is discontinuous or changes in type, the integral point may represent either one of its neighboring boundary conditions, depending on which one contains the point. This may cause a serious problem if the angular integral points are too few, or the jumps in boundary conditions are too large, or the medium is optically thin where the contribution of the boundary to radiation is important.

A simple remedy to this problem is to apply more angular integral points for only the boundary integrations. This strategy does not increase significantly the computing time because the calculation of the boundary integrals is a small portion compared with the volume integrations. Another more sophisticated remedy would be using adaptive schemes for the boundary integrations according to the boundary conditions.

The numerical inaccuracy mainly results from discretization error and numerical integration error. In the YIX method, those two errors are independent, because the selection of the integration points is unrelated to the grid. This is an advantage of the YIX method in solving combined heat-transfer problems, where we can use a dense grid to handle the complex flowfield while applying relatively sparse integration points to treat the generally smoother radiation field.

Two-Dimensional Problem in a Square Enclosure: Inhomogeneous Medium

The problem is the same as above, except that the extinction coefficient is not uniform in the medium (Figs. 4c and 4d). To

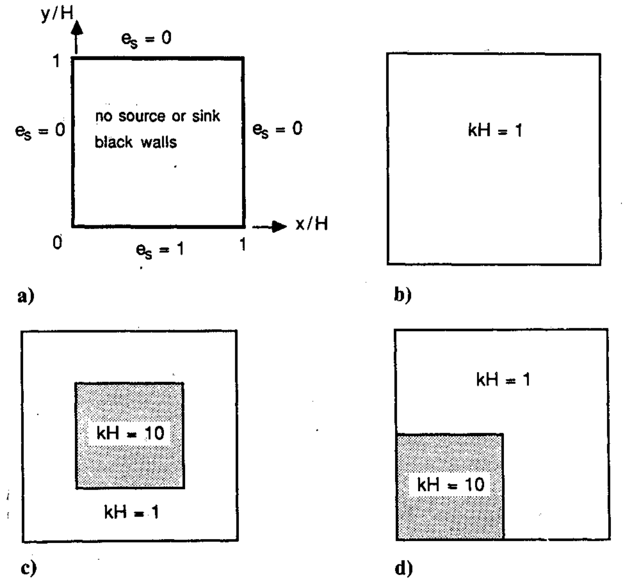

Fig. 4 Geometry and optical properties for two-dimensional radiation in a square enclosure: a) geometry and boundary conditions; b) extinction coefficient distribution for case 1; c) extinction coefficient distribution for case 2; and d) extinction coefficient distribution for case 3.

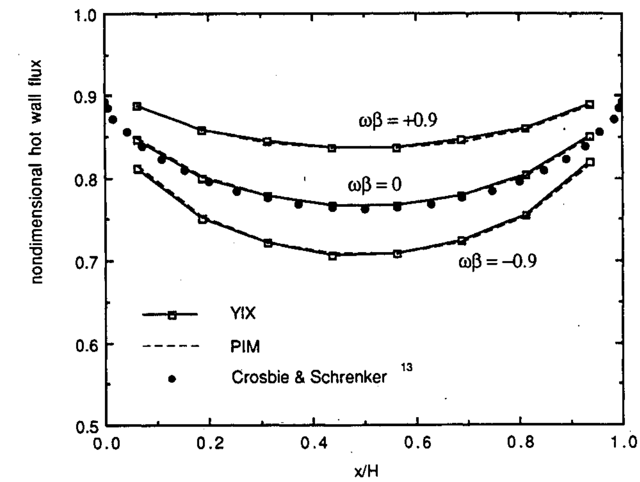

Fig. 5 Comparative nondimensional hot wall flux for radiation transfer in a two-dimensional square enclosure. Influence of anisotropic scattering.

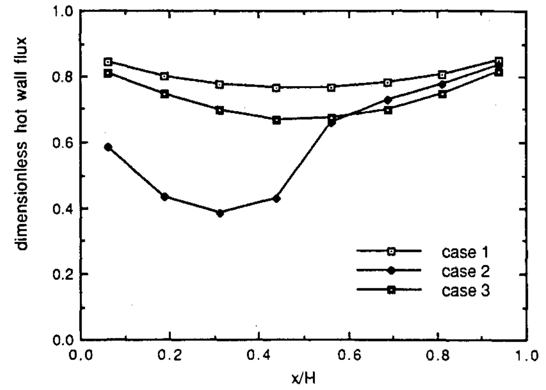

Fig. 6 Comparative dimensionless hot wall flux for radiation transfer in a two-dimensional square enclosure in cases 1, 2, and 3.

study the influence of extinction coefficient distributions, three cases are considered:

1) $kH = 1$ in $0 \le x/H, y/H \le 1$;

2) $kH = 10$ in $0.25 \le x/H, y/H \le 0.75$; and $kH = 1$ in other regions;

3) $kH = 10$ in $0 \le x/H, y/H \le 0.5$; and $kH = 1$ in other regions.

Figure 6 shows the radiation flux on the hot (lower) wall. In both cases 2 and 3, a highly absorbing region is inserted in the medium, so the "resistance" of the medium to radiation is increased. This decreases the total radiation flux, as seen in the figure. In case 2, because the highly absorbing region is at the center, the greatest reduction in heat flux compared with case 1 is found at $x/H = 0.5$. Similarly, the minimum hot wall flux for case 3 is found near $x/H = 0.25$, where the centerline of the high absorbing lump lies. Note that the slip boundary condition at the lower left corner in this case enables the flux to be quite large near $x/H = 0$. When $x/H > 0.6$, the influence of the inhomogeneity is small and only a slight reduction in hot wall flux is found.

Conclusions

The YIX method, which solves numerically the radiation transport equations of integral form, has been described in this paper. Some one-dimensional and two-dimensional example problems were studied and results discussed. The following conclusions are drawn.

1) The YIX method is efficient compared with other methods of solving the radiation transport integral equations. In the limit where an infinite number of integration points is used, it yields the same results as the product integration method.

2) Unlike the zonal method and the product integration method, the YIX method can easily deal with inhomogeneous radiation properties in the medium. This enables it to solve real-gas radiation problems.

3) In some cases, inaccuracy can be induced by a phenomenon similar to the ray effect of discrete ordinates method. The inaccuracy can be cured by simple remedies.

4) As is the case for the zonal method and the product integration methods, the YIX method can be used easily to solve combined radiation and conduction/convection problems. One application can be found in Tan and Howell.[11] Because the distribution of integration points is independent of the mesh, a large number of grid points can be used to handle the complex flowfield, while the computer time for the radiation portion using the same mesh does not increase rapidly.

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